

Complete Analysis on the Short Distance Contribution of $B_s \rightarrow \ell^+ \ell^- \gamma$ in Standard Model

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Abstract

Using the B_s meson wave function extracted from non-leptonic B_s decays, we evaluate the short distance contribution of rare decays $B_s \rightarrow \ell^+ \ell^- \gamma$ ($\ell = e, \mu$) in the standard model, including all the possible diagrams. We focus on the contribution from four-quark operators which are not taken into account properly in previous researches. We found that the contribution is large, leading to the branching ratio of $B_s \rightarrow \ell^+ \ell^- \gamma$ being nearly enhanced by a factor 3 and up to 1.7×10^{-8} . The predictions for such processes can be tested in the LHC-b and B factories in near future.

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I. INTRODUCTION

The standard model (SM) of electroweak interaction has been remarkably successful in describing physics below the Fermi scale and is in good agreement with the most experiment data. One of the most promising processes for probing the quark-flavor sector of the SM is the rare decays. These decays, induced by the flavor changing neutral currents (FCNC) which occur in the SM only at loop level, play an important role in the phenomenology of particle physics and in searching for the physics beyond the SM [1, 2]. The observation of the penguin-induced decay $B \rightarrow X_s \gamma$, $B \rightarrow X_s \ell^+ \ell^-$ ($\ell = e, \mu$) are in good agreement with the SM prediction, and the first evidence for the decay $B_s \rightarrow \mu^+ \mu^-$ was confirmed at the end of 2012 [3], putting strong constraints on its various extensions. Nevertheless, these processes are also important in determining the parameters of the SM and some hadronic parameters in QCD, such as the CKM matrix elements, the meson decay constant f_{B_s} , providing information on heavy meson wave functions.

Thanks to the Large Hadron Collider (LHC) at CERN we have entered a new era of particle physics. In experimental side, in the current early phase of the LHC era, the exclusive modes such as $B_s \rightarrow \ell^+ \ell^- \gamma$ ($\ell = e, \mu$) are among the most promising decays due to their relative cleanliness and sensitivity to models beyond the SM [1, 2]. In theoretically side, since no helicity suppression exists and large branching ratios as $B_s \rightarrow \mu^+ \mu^-$ are expected. There are mainly two kinds of contributions of $B_s \rightarrow \ell^+ \ell^- \gamma$ in the SM: the short distance contribution which can be evaluated reliably by perturbation theory [4] while the long distance QCD effects describing the neutral vector-meson resonances ϕ and J/Ψ family [5–7]. As for the short distance contribution, it is thought in previous works that a necessary work is only attaching real photon to any charged internal and external lines in the Feynman diagrams of $b \rightarrow s \ell^+ \ell^-$ with statement that contributions from the attachment of photon to any charged internal propagator are regraded as to be strongly suppressed and can be neglected safely [1, 2, 8, 9], i.e., one can easily obtain the amplitude of $B_s \rightarrow \ell^+ \ell^- \gamma$ by using the effective weak Hamiltonian of $b \rightarrow s \ell^+ \ell^-$ and the matrix elements $\langle \gamma | \bar{s} O_i b | B_s \rangle$ $O_i = \gamma_\mu P_L, \sigma_{\mu\nu} q^\nu P_R$ directly. Therefore contributions from the attachment of real photon with magnetic-penguin vertex to any charged external lines are of course omitted [1, 2] or stated to be negligibly small [9]. Another contribution from loop insertion of the lower order four-quark operators are also always neglected. We note that the complete contribution

seems to have been done in [5], however it mainly concentrated on the long distance effects of the meson resonances, whereas the short distance contribution was indeed incompletely analyzed. A complete examination included all contribution to the processes in the SM is needed.

As being well known, only short-distance contribution can be reliably predicted, and it is more important than the long-distance contribution from the resonances which is actually excluded partly by setting cuts in experimental measurements. Recently we showed that the contributions from the attachment of real photon with magnetic-penguin vertex to any charged external lines can enhance the branching ratios of $B_s \rightarrow \ell^+ \ell^- \gamma$ by a factor about 2 [10].

In this letter, we will extend our previous studies and use the B_s meson wave function extracted from non-leptonic B_s decays [11] to reevaluate the short distance contribution from the all categories of diagrams of $B_s \rightarrow \ell^+ \ell^- \gamma$ decays. Special attention will be payed on the contribution from the four-quark operators, and a comparative study with previous work will be discussed. The paper are organized in the following, in sec. II, we analyse the full short distance contribution and present detail calculation of exclusive decays $B_s \rightarrow \ell^+ \ell^- \gamma$. The numerical results and the comparative study are given in sec. III, and the conclusions are given in sec. IV.

II. COMPLETE ANALYSIS ON SHORT DISTANCE CONTRIBUTIONS

In order to simplify the decay amplitude for $B_s \rightarrow \ell^+ \ell^- \gamma$, we have to utilize the B_s meson wave function, which is not known from the first principal and model depended. Fortunately, many studies on non-leptonic B [12, 13] and B_s decays [11] have constrained the wave function strictly. It was found that the wave function has form

$$\Phi_{B_s} = (\not{p}_{B_s} + m_{B_s}) \gamma_5 \phi_{B_s}(x), \quad (1)$$

where the distribution amplitude $\phi_{B_s}(x)$ can be expressed as [14]:

$$\phi_{B_s}(x) = N_{B_s} x^2 (1-x)^2 \exp\left(-\frac{m_B^2 x^2}{2\omega_{b_s}^2}\right) \quad (2)$$

with x being the momentum fractions shared by s quark in B_s meson. The normalization constant N_{B_s} can be determined by comparing

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = i N_c \int_0^1 \phi_{B_s}(x) dx \text{Tr} [\gamma^\mu \gamma_5 (\not{p}_{B_s} + m_{B_s}) \gamma_5] dx = -4 N_c i p_{B_s}^\mu \int_0^1 \phi_{B_s}(x) dx \quad (3)$$

with N_c being the number of quarks and

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = -i f_{B_s} p_{B_s}^\mu, \quad (4)$$

the B meson decay constant f_{B_s} is thus determined by the condition

$$\int_0^1 \phi_{B_s}(x) dx = \frac{1}{4 N_c} f_{B_s}. \quad (5)$$

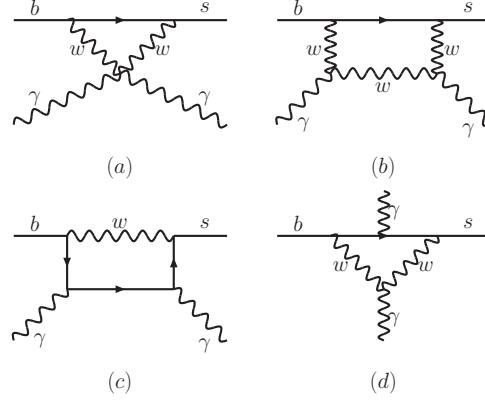


FIG. 1: Feynman diagrams without effective vertex of $b \rightarrow s \gamma$ contribution to $B_s \rightarrow \gamma \gamma$.

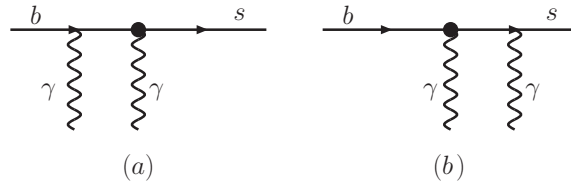


FIG. 2: Feynman diagrams with effective vertex of $b \rightarrow s \gamma$ contribution to $B_s \rightarrow \gamma \gamma$. The black dot stands for the magnetic-penguin operator O_7 .

Let us start with the quark level processes $B_s \rightarrow \ell^+ \ell^- \gamma$ which are subject to the QCD corrected effective weak Hamiltonian. The general effective Hamiltonian that describes

$b \rightarrow s$ transition is given by

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{j=1}^{10} C_j(\mu)O_j(\mu), \quad (6)$$

where O_j ($j = 1, \dots, 6$) stands for the four-quark operators, and the forms and the corresponding Wilson coefficients C_i can be found in Ref. [15].

Generally, to describe all the short distance of the process $B_s \rightarrow \ell^+\ell^-\gamma$, new effective operators for $b \rightarrow s\gamma\gamma$ which are not included in (6) should be introduced. Corresponding feynman diagrams without and with effective vertex $b \rightarrow s\gamma$ are shown in FIG. 1 and FIG. 2, respectively. When connect di-lepton line to one γ , operator $b \rightarrow s\gamma\gamma$ may contribute to $B_s \rightarrow \ell^+\ell^-\gamma$. Contributions from the such kind of diagrams with a photon attaching from internal charged lines to $B_s \rightarrow \ell^+\ell^-\gamma$ are usually regraded as to be strongly suppressed by a factor m_b^2/m_W^2 thus can be neglected safely [1, 2, 8, 9]. However, as pointed out in [16], the conclusion of this is correct, but the explanation is not as what it is described. Here we address the reason more clearly: the contribution from diagrams FIG. 1 (a) and FIG. 1 (b) are not suppressed. When applying effective vertex of $b \rightarrow s\gamma$ to describe $b \rightarrow s\gamma\gamma$ as shown in FIG. 2, internal quarks in the effective vertex are off-shell and such off-shell effects are also not suppressed. We have proven that the such two non-suppressed effects in FIG. 1 and FIG. 2 cancel each other exactly [16]. Therefore we can use the effective operators listed in Eq. (6) for on-shell quarks to calculate the total short distance contributions of $B_s \rightarrow \ell^+\ell^-\gamma$ in SM safely.

The Feynmann diagrams contributing to $B_s \rightarrow \ell^+\ell^-\gamma$ at parton level can then be classified into three kinds as follows:

1. Attaching a real photon to any charged external lines in the Feynman diagrams of $b \rightarrow s\ell^+\ell^-$;
2. Attaching a virtual photon to any charged external lines in the Feynman diagrams of $b \rightarrow s\gamma$ with virtual photon into lepton pairs;
3. Attaching two photon to any charged external lines in the Feynman diagrams of four-quark operators with one of two photon into lepton pairs.

Note that the third contribution is not considered in the previous studies except for Ref. [5] which is the focus of this paper and will show the detail in the following. We also will discuss these contributions seperatly.

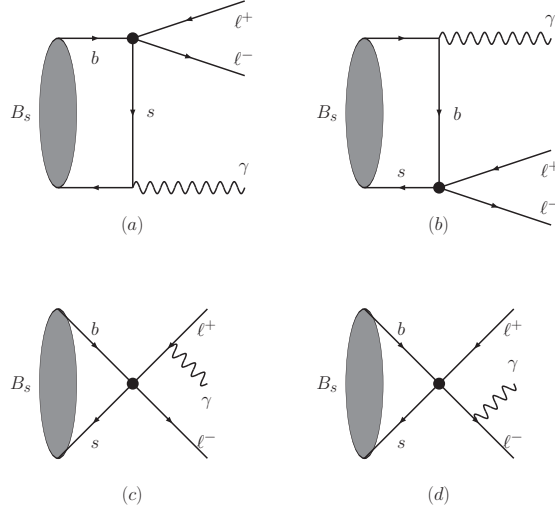


FIG. 3: Feynman diagrams that contribute to the matrix elements $B_s \rightarrow \ell^+ \ell^- \gamma$ with the contribution of O_7 , O_9 , O_{10} in tree level.

A. External real photon contributions

The Feynman diagrams of the first kind of contributions are shown in FIG. 3. As the contribution from the FIG. 3 (c) and (d) with photon attached to external lepton lines, considering the fact that (i) being a pseudoscalar meson, B_s meson can only decay through axial current, so the magnetic penguin operator O_7 's contribution vanishes; (ii) the contribution from operators O_9 , O_{10} has the helicity suppression factor m_ℓ/m_{B_s} , so for light lepton electron and muon, we can neglect their contribution safely. These diagrams in FIG. 3 (a) and (b) are always regarded as the dominant contributions, and they have been considered by using the light cone sum rule [1, 2], the simple constituent quark model [8], and the B meson distribution amplitude extracted from non-leptonic B decays [9]. We rewrite the amplitude of $B_s \rightarrow \ell^+ \ell^- \gamma$ at meson level as [10]:

$$\begin{aligned}
 A_I = iN_c G e e_d \frac{1}{p_{B_s} \cdot k} & \left\{ [C_1 i \epsilon_{\alpha\beta\mu\nu} p_{B_s}^\alpha \varepsilon^\beta k^\nu + C_2 p_{B_s}^\nu (\varepsilon_\mu k_\nu - k_\mu \varepsilon_\nu)] \bar{\ell} \gamma^\mu \ell \right. \\
 & \left. + C_{10} [C_+ i \epsilon_{\alpha\beta\mu\nu} p_{B_s}^\alpha \varepsilon^\beta k^\nu + C_- p_{B_s}^\nu (\varepsilon_\mu k_\nu - k_\mu \varepsilon_\nu)] \bar{\ell} \gamma^\mu \gamma_5 \ell \right\}. \quad (7)
 \end{aligned}$$

The form factors in Eq. (7) are found to be:

$$\begin{aligned} C_1 &= C_+ \left(C_9^{eff} - 2 \frac{m_b m_{B_s}}{q^2} C_7^{eff} \right), \\ C_2 &= C_9^{eff} C_- - 2 \frac{m_b m_{B_s}}{q^2} C_7^{eff} C_+, \end{aligned} \quad (8)$$

with the constant $G = \alpha_{em} G_F V_{tb} V_{ts}^* / (\sqrt{2}\pi)$, and

$$C_{\pm} = \int_0^1 \left(\frac{1}{x} \pm \frac{1}{y} \right) \phi_{B_s}(x) dx. \quad (9)$$

The expression in Eq. (7) can be compared with Ref. [9].

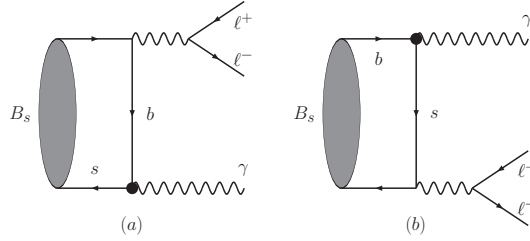


FIG. 4: Feynman diagrams of $b \rightarrow s \gamma$ with virtual photon into lepton pairs .

B. External virtual photon contributions

The Feynman diagrams of the second kind of contributions are shown in FIG. 4. Contributions from the kind of diagrams are always neglected [1, 2] or stated to be negligibly small [9]. Note the B_s meson wave functions used in this work and Ref. [9] are both from non-leptonic B_s decays. However, as mentioned in the introduction the authors of Ref. [9] did not present the expression of the contribution from FIG. 4 and only stated the contribution is numerical negligibly small. But such statement seems to be questionable, for that the pole of propagator of the charged line attached by photon may enhance the decay rate greatly which make some diagrams can not be neglected in the calculation. In these two diagrams, photon of the magnetic-penguin operator is real, thus its contribution to $B_s \rightarrow \ell^+ \ell^- \gamma$ is different from the first kind contributions. We get the amplitude [10]:

$$A_{II} = i 2 N_c G e e_d C_7^{eff} \frac{m_b m_{B_s}}{q^2} \frac{1}{p_{B_s} \cdot q} \overline{C}_+ [k_\mu q \cdot \epsilon - \epsilon_\mu k \cdot q - i \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu k^\alpha q^\beta] [\bar{\ell} \gamma^\mu \ell], \quad (10)$$

with coefficients \overline{C}_+ obtained by a replacement:

$$\begin{aligned}\overline{C}_+ &= C_+(x \rightarrow \bar{x} = x - z - i\epsilon; y \rightarrow \bar{y} = y - z - i\epsilon) \\ &= N_B \int_0^1 dx \left(\frac{1}{x - z - i\epsilon} + \frac{1}{1 - x - z - i\epsilon} \right) x^2 (1 - x)^2 \exp \left[-\frac{m_{B_s}^2}{2\omega_{B_s}^2} x^2 \right],\end{aligned}\quad (11)$$

where $z = \frac{q^2}{2p_{B_s} \cdot q}$ and the first, second term in (11) denotes the contribution from FIG. 4 (a) and (b) respectively. Note that the contribution from FIG. 3 (a) is much larger than (b) since $m_{B_s} \ll \omega_{B_s}$ (see next section) which is very easily understood in simple constituent quark model [8], i.e., $\phi_{B_s}(x) = \delta(x - m_s/m_{B_s})$. However, the contributions from Fig.4 (a) and (b) are comparable, and pole in \overline{C}_+ corresponds to the pole of the quark propagator when it is connected by the off-shell photon propagator. Thus the \overline{C}_+ term may enhance the decay rate of $B_s \rightarrow \ell^+ \ell^- \gamma$ and its analytic expression reads

$$\begin{aligned}\overline{C}_+ &= 2N_{B_s} \pi i z^2 (1 - z)^2 \exp \left[-\frac{m_{B_s}^2}{2\omega_{B_s}^2} z^2 \right] \\ &+ N_{B_s} \int_0^1 dx \left(\frac{1}{x + z} - \frac{1}{1 + x - z} \right) x^2 (1 + x)^2 \exp \left[-\frac{m_{B_s}^2}{2\omega_{B_s}^2} x^2 \right] \\ &- N_{B_s} \int_{-1}^1 \left(\frac{1}{\frac{1}{x} - z} + \frac{1}{1 - \frac{1}{x} - z} \right) \frac{dx}{x^4} \left(1 - \frac{1}{x} \right)^2 \exp \left[-\frac{m_{B_s}^2}{2\omega_{B_s}^2} \frac{1}{x^2} \right].\end{aligned}\quad (12)$$

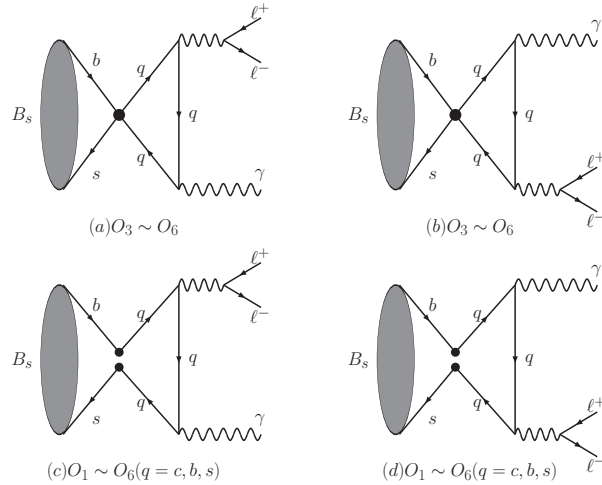


FIG. 5: Feynman diagrams that contribute to the process $B_s \rightarrow \ell^+ \ell^- \gamma$ with possible insertion of O_1 to O_6 in loop level .

C. Quark weak annihilation contributions

Now we focus on the contributions from the diagrams for the four-quark operators which are not considered properly in previous works. The Feynman diagrams of the third kind of contribution are shown in FIG. 5. The operator O_7 is high order than four-quark operator $O_1 - O_6$, thus the contribution of loop diagram with operator $O_1 - O_6$ insertion should be comparable with the tree level electro-weak penguin $O_7 - O_{10}$ contributions listed above. To calculate the leading order matrix elements of $b \rightarrow s\gamma^*\gamma$ shown in FIG. 5, we express the decay amplitude for $b(p_b) \rightarrow s(p_s)\gamma^*(k_1)\gamma(k_2)$ as

$$A_{\text{III}}(b \rightarrow s\gamma^*\gamma) = i\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{e^2}{16\pi^2}\bar{s}(p_s)T_{\mu\nu}b(p_b)\epsilon^\mu(k_1)\epsilon^\nu(k_2), \quad (13)$$

where $p_{b,s}, k_{1,2}$ denotes the momentum of quarks and photon respectively, ϵ is the vector polarization of photon. We split the tensor $T^{\mu\nu}$ into the momentum odd and even power terms for simplification. Keeping our physics goal in mind, Without loss of generality we assume that photon with momentum k_1 is virtual and drop the terms proportional to k_2^ν in the expressions. After straight calculation, we obtain

$$T_{\mu\nu}^{\text{odd}}(q) = e_q^2 \left\{ \frac{1}{k_1 k_2} \left[i\epsilon_{\nu\alpha\beta\lambda}\gamma^\lambda\gamma_5 k_1^\alpha k_2^\beta [k_{1\mu}f_2(q) - k_{2\mu}f_+(q)] + i\epsilon_{\mu\alpha\beta\lambda}\gamma^\lambda\gamma_5 k_1^\alpha k_2^\beta k_{1\nu}f_+(q) \right] \right. \\ \left. + i\epsilon_{\mu\nu\alpha\beta}\gamma^\beta\gamma_5 [k_1^\alpha f_+(q) - k_2^\alpha f_-(q)] \right\}, \quad (14)$$

$$T_{\mu\nu}^{\text{even}}(q) = -2\frac{e_q^2}{m_q} \left\{ (k_{1\nu}k_{2\mu} - k_1k_2g_{\mu\nu}) \left[f_3(q) + \left(1 - \frac{4}{z_q}\right)f_1'(q) \right] + i\epsilon_{\mu\nu\alpha\beta}\gamma_5 k_1^\alpha k_2^\beta f_1'(q) \right\} \quad (15)$$

where q denotes the quark in the internal line which two photons are attached and e_q is the number of electrical charge of the quark. The loop functions appearing in (15) have forms as

$$f_\pm(q) = \frac{1}{2} + \frac{1}{z_q} \int_0^1 \ln g(z_q, u_q, x) \frac{dx}{x} \mp \frac{u_q}{2z_q} \int_0^1 \ln g(z_q, u_q, x), \\ f_1(q) = \frac{1}{2}[f_+(q) + f_-(q)], \quad f_2(q) = \frac{z_q}{2u_q}[f_+(q) - f_-(q)], \\ f_3(q) = \frac{2}{z_q} - \frac{2u_q}{z_q^2} \int_0^1 \ln g(z_q, u_q, x) dx, \\ g(z, u, x) = \frac{1 - (u + z)x(1 - x)}{1 - ux(1 - x)}, \quad (16)$$

with $f_1'(q) = \frac{1}{2} - f_1(q)$, $z_q = \frac{2k_1 \cdot k_2}{m_q^2}$ and $u_q = \frac{k_1^2}{m_q^2}$. Writing the amplitude in this ways, one can infer easily that for example, when operators O_j ($j = 1, \dots, 4$) are inserted, only $T_{\mu\nu}^{\text{odd}}$

part can contribute to $b \rightarrow s\gamma\gamma$ while the process receives both parts contributions as $O_{5,6}$ are inserted. It is also easily obtained the similar result for the on-shell photons as in Ref. [17] by setting $u_q = 0$.

With the amplitude of $b \rightarrow s\gamma^*\gamma$ and B_s wave function ready, we write the total contribution from FIG. 5 to exclusive decay of $B_s(p_{B_s}) \rightarrow \gamma(k)\ell^+\ell^-$ as

$$A_{\text{III}} = -2ie \frac{f_{B_s} G}{q^2} \sum_{j=1}^6 C_j(m_b) [T_1^j p_{B_s}^\nu (\epsilon_\nu k_\mu - \epsilon_\mu k_\nu) + T_2^j i \epsilon_{\mu\nu\alpha\beta} p_{B_s}^\alpha k^\beta \epsilon^\nu] [\bar{\ell} \gamma^\mu \ell], \quad (17)$$

with the form factors given by

$$\begin{aligned} T_1^1 &= T_1^2 = T_1^3 = T_1^4 = 0; \\ T_2^1 &= N_c T_2^2 = N_c e_u^2 f_1(\bar{z}_c, t); \\ T_2^3 &= N_c \left\{ \sum_{q=u,d,s,c,b} e_q^2 f_1(\bar{z}_q, t) + e_d^2 [f_1(\bar{z}_b, t)) + f_1(\bar{z}_s, t)] \right\} \\ T_2^4 &= \sum_{q=u,d,s,c,b} e_q^2 f_1(\bar{z}_q, t) + e_d^2 N_c [f_1(\bar{z}_b, t)) + f_1(\bar{z}_s, t)] \\ T_1^5 &= \frac{1}{N_c} T_1^6 = 2e_d^2 \left\{ \frac{1}{\bar{z}_{b^{1/2}}} \left[f_3(\bar{z}_b, t) + (1 - \frac{4\bar{z}_b}{1-t}) (\frac{1}{2} - f_1(\bar{z}_b, t)) \right] - (b \rightarrow s) \right\}; \\ T_2^5 &= -N_c \sum_{q=u,d,s,c,b} e_q^2 f_1(\bar{z}_q, t) - 2e_d^2 \left[\frac{1}{\bar{z}_{b^{1/2}}} (\frac{1}{2} - f_1(\bar{z}_b, t)) + (b \rightarrow s) \right]; \\ T_2^6 &= - \sum_{q=u,d,s,c,b} e_q^2 f_1(\bar{z}_q, t) - 2N_c e_d^2 \left[\frac{1}{\bar{z}_{b^{1/2}}} (\frac{1}{2} - f_1(\bar{z}_b, t)) + (b \rightarrow s) \right], \end{aligned} \quad (18)$$

where q^2 in Eq. (17) is the invariant mass square of lepton pair. The functions can be obtained directly from (16) by redefined parameters $\bar{z}_q = m_q^2/m_{B_s}^2$, $t = q^2/m_{B_s}^2$,

$$f_1(\bar{z}, t) = \frac{1}{2} + \frac{\bar{z}}{1-t} \int_0^1 \frac{dx}{x} \ln \left[\frac{\bar{z} - x(1-x)}{\bar{z} - tx(1-x)} \right], \quad (19)$$

$$f_3(\bar{z}, t) = \frac{2\bar{z}}{1-t} \left\{ 1 - \frac{t}{1-t} \int_0^1 dx \ln \left[\frac{\bar{z} - x(1-x)}{\bar{z} - tx(1-x)} \right] \right\}, \quad (20)$$

with explicit formula needed in calculation given by

$$\begin{aligned} \int_0^1 \frac{dy}{y} \ln[1 - vy(1-y) - i\epsilon] &= \begin{cases} -2\arctan^2 \sqrt{\frac{v}{4-v}} & \text{for } v < 4; \\ -\frac{\pi^2}{2} - 2i\pi \ln \frac{\sqrt{v} + \sqrt{v-4}}{2} + 2 \ln^2 \frac{\sqrt{v} + \sqrt{v-4}}{2} & \text{for } v > 4, \end{cases} \\ \int_0^1 dy \ln[1 - vy(1-y) - i\epsilon] &= -2 + |1-x|^{1/2} \begin{cases} \ln \left| \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} \right| - i\pi & \text{for } x = 4/v < 1; \\ 2 \arctan \frac{1}{\sqrt{x-1}} & \text{for } x = 4/v > 1. \end{cases} \end{aligned} \quad (21)$$

From Eq. (17) it is clear that the contribution from four-quark operators to $B_s \rightarrow \gamma \ell^+ \ell^-$ has the similar expression as that from magnetic-penguin operator with real photon to $B_s \rightarrow \ell^+ \ell^- \gamma$. Thus the total matrix element for the decay $B_s \rightarrow \ell^+ \ell^- \gamma$ including contributions from three kinds of diagrams can be obtained easily by a shift to the form factors:

$$\overline{C}_1 = C_+ \left[C_9^{eff} - \frac{2m_b m_{B_s}}{q^2} C_7^{eff} \left(1 + \frac{p_{B_s} \cdot k}{p_{B_s} \cdot q} \right) \right] - 2 \frac{p_{B_s} \cdot k}{q^2} \frac{f_{B_s}}{e_d} \sum_{j=1}^6 C_j T_2^j, \quad (22)$$

$$\overline{C}_2 = C_9^{eff} C_- - \frac{2m_b m_{B_s}}{q^2} C_7^{eff} C_+ \left(1 + \frac{p_{B_s} \cdot k}{q^2} \right) + 2 \frac{p_{B_s} \cdot k}{q^2} \frac{f_{B_s}}{e_d} \sum_{j=1}^6 C_j T_1^j. \quad (23)$$

Finally, we get the differential decay width versus the photon energy E_γ ,

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{em}^3 G_F^2}{108\pi^4} |V_{tb} V_{ts}^*|^2 (m_{B_s} - 2E_\gamma) E_\gamma \left[|\overline{C}_1|^2 + |\overline{C}_2|^2 + C_{10}^2 (|C_+|^2 + |C_-|^2) \right]. \quad (24)$$

III. RESULTS AND DISCUSSIONS

The decay branching ratios can be easily obtained by integrating over photon energy. In the numerical calculations, we use the following parameters [19]:

$$\alpha_{em} = \frac{1}{137}, \quad G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}, \quad m_b = 4.2 \text{GeV},$$

$$|V_{tb}| = 0.999, \quad |V_{ts}| = 0.04, \quad |V_{td}| = 0.0084$$

$$m_{B_s} = 5.37 \text{GeV}, \quad \omega_{B_s} = 0.5, \quad f_{B_s} = 0.24 \text{GeV}, \quad \tau_{B_s} = 1.47 \times 10^{-12} \text{s}.$$

$$m_{B_d^0} = 5.28 \text{GeV}, \quad \omega_{B_d} = 0.4, \quad f_{B_d} = 0.19 \text{GeV}, \quad \tau_{B_d} = 1.53 \times 10^{-12} \text{s}.$$

The ratios of $B_s \rightarrow \gamma \ell^+ \ell^-$ are shown in Table I together with results of $B_{d,s} \rightarrow \gamma \ell^+ \ell^-$ from this work and our previous research for comparison. The errors shown in the Table I comes from the heavy meson wave function, by varying the parameter $\omega_{B_d} = 0.4 \pm 0.1$, and $\omega_{B_s} = 0.5 \pm 0.1$ [9]. Note that, the predicted branching ratios receive errors from many parameters, such as parameters meson decay constant, meson and quark masses.

From the numerical results we conclude that unlike in decay $B \rightarrow X_s \gamma \gamma$ where the four-quark operators just contribute a few percent to the branching ratio [17], our numerical result shows that contribution from the four-quark operators to $B_s \rightarrow \gamma \ell^+ \ell^-$ is large. It can be understood as follows:

1. As pointed out in Ref. [9], the radiative leptonic decays are very sensitive probes in extracting the heavy meson wave functions;

TABLE I: Comparison of branching ratios in unit of 10^{-9} with previous calculations

Branching Ratios ($\times 10^{-9}$)	Results		
	The first kind of diagrams	The first two kind of diagrams	Included all diagrams
$B_s \rightarrow \gamma \ell^+ \ell^-$	$5.16^{+2.42}_{-1.38}$	$10.2^{+4.11}_{-2.51}$	$17.36^{+4.55}_{-2.63}$
$B_d^0 \rightarrow \gamma \ell^+ \ell^-$	$0.21^{+0.14}_{-0.06}$	$0.40^{+0.26}_{-0.13}$	$0.53^{+0.26}_{-0.12}$

2. Values of the Wilson coefficients $C_j(m_b)$ ($j = 3, \dots, 6$) are at order of 10^{-2} , indicating that contribution to the corresponding operators via T^j is less important compared with those from $O_{1,2}$;
3. From Eq. (18), one can infer easily that the four-quarks contribution to the form factors in (23) have coefficient $(N_c C_1 + C_2) T_2^2 f_{B_s} / e_d$ while the contribution from magnetic-penguin operator with real photon, $m_b m_{B_s} / (p_{B_s} \cdot q) C_7^{eff} C_+$. Note $(N_c C_1 + C_2) / e_d$ and C_7^{eff} can be comparable and have the same sign in \overline{C}_1 and opposite sign in \overline{C}_2 . However, with $T_1^j = 0$ for $j = 1, 2$ thus comparable contribution studied in this work and in Ref. [10] is expected, leading to enhancement of branching ratios of $B_s \rightarrow \gamma \ell^+ \ell^-$ when new diagrams are taken into account.
4. The predicted short-distance contributions from quark weak annihilation as well as the magnetic-penguin operator with real photon to the exclusive decay are large, and the branching ratios of $B_s \rightarrow \ell^+ \ell^- \gamma$ are enhanced nearly by a factor 3 compared with that only contribution from magnetic-penguin operator with virtual photon and up to 1.7×10^{-8} , implying the search of $B_s \rightarrow \ell^+ \ell^- \gamma$ can be achieved in near future.
5. Due to the large contributions from magnetic-penguin operator with real photon and quark weak annihilation, the form factors for matrix elements $\langle \gamma | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_s \rangle$ and $\langle \gamma | \bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu b | B_s \rangle$ as a function of dilepton mass squared q^2 are complex and not as simple as $1/(q^2 - q_0^2)^2$ where q_0^2 is constant [18]. The $B_s \rightarrow \gamma$ transition form factors predicted in this works have also some differences from those in Ref. [5–7]. For instance, Ref. [6] predicted the form factors $F_{TV}(q^2, 0)$, $F_{TA}(q^2, 0)$ induced by tensor and pseudotensor currents with direct emission of the virtual photon from quarks are

only equal at maximum photon energy, whereas the corresponding formula in this work have the same expression as $-\frac{e_d N_c m_{B_s}}{p_{B_s} \cdot k} C_+ \propto 1/(q^2 - q_0^2)$ in Eq. (7). Furthermore, the form factors are larger than previous predictions.

To clarify things more clear, we think it is necessary to present a few more comments about the calculation of Ref. [5], as mentioned in introduction. In order to estimate the contribution of direct emission of the real photon from quarks, the authors of Ref. [5] calculated the form factors $F_{TA,TV}(0, q^2)$ by including the short-distance contribution in $q^2 \rightarrow 0$ limit and additional long-distance contribution from the resonances of vector mesons such as ρ^0 , ω^0 for B_d decay and ϕ for B_s decay. Obviously, this means the short-distance contributions were not appropriately taken into account. Moreover, if $F_{TA,TV}(0, q^2) = F_{TA,TV}(0, 0)$ stands for the short distance contribution, it seems to double counting since in this case photons emitted from magnetic-penguin vertex and quark lines directly are not able to be distinguished.

We also note that for contribution from the weak annihilation the authors of Ref. [5] only took into account u and c quarks in the loop by axial anomaly as the long distance contribution, they concluded that the anomalous contribution is suppressed by a power of a heavy quark mass. We believe that only anomalous contribution to account contribution from the weak annihilation is insufficient. Our numerical result shows that the contributions from weak-annihilation diagrams are large and can not be neglected.

IV. CONCLUSION

In summary, we evaluated short distance calculation of the rare decays $B_s \rightarrow \gamma \ell^+ \ell^-$ in the SM, including contributions from all four kinds of diagrams. We focus on the contribution from four-quark operators which are not taken into account properly in previous researches. We found that the contributions are large, leading to the branching ratio of $B_s \rightarrow \ell^+ \ell^- \gamma$ being nearly enhanced by a factor 3. In the current early phase of the LHC era, the exclusive modes with muon final states are among the most promising decays. Although there are some theoretical challenges in calculation of the hadronic form factors and non-factorable corrections, with the predicted branching ratio at order of 10^{-8} , $B_s \rightarrow \mu^+ \mu^- \gamma$ can be expected as the next goal after $B_s \rightarrow \mu^+ \mu^-$ since the final states can be identified easily and the branching ratios are large. Experimentally, $B_s \rightarrow \mu^+ \mu^- \gamma$ mode is one of the main

backgrounds to $B_s \rightarrow \mu^+\mu^-$, and thus it is already taken into account in $B_s \rightarrow \mu^+\mu^-$ searches [3]. Our predictions for such processes can be tested in the LHC-b and B factories in near future.

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